Exercise 1.1

Q1 Over the last few years Jules has had a 90% success rate in germinating her geranium plants. This year she has bought an improved variety of seeds and hopes for even better results.

a) Which quantity is Jules investigating?

b) What value has this quantity taken over the last few years?

c) Write down a suitable: (i) null hypothesis, (ii) alternative hypothesis.

d) State whether this test is one- or two-tailed.

Q2 The local council found that only 16% of residents were aware that grants were available to help pay to insulate their houses. The council ran a campaign to publicise the grants, and now want to test whether there is an increased awareness in the area. Write down suitable null and alternative hypotheses involving the proportion of residents aware of the grants.

Q3 In a village shop, 3% of customers buy a jar of chilli chutney. The owner has changed the packaging of the chutney and wants to know if the proportion of customers buying a jar of chilli chutney has changed. Write down suitable null and alternative hypotheses.

Q4 It is claimed that the proportion of members of a gym who watch Australian soaps is 40%. Boyd wants to test his theory that the proportion is higher. Write down suitable null and alternative hypotheses.

To set up a hypothesis test for a **binomial** distribution:

1. Define the **population parameter in context** — for a binomial distribution it's always p, a **probability** of success, or **proportion** of a population.
2. Write down the **null** hypothesis (H_0) — H_0: $p = a$ for some constant a.
3. Write down the **alternative** hypothesis (H_1) — H_1 will either be $H_1: p < a$ or $H_1: p > a$ (one-tailed test) or $H_1: p \neq a$ (two-tailed test).
4. State the **test statistic**, X.

   A **test statistic** for a hypothesis test is a statistic calculated from **sample data**, which is used to **decide** whether or not to reject H_0.

   For a binomial distribution it's always the **number of 'successes'** in the sample.
5. Write down the **probability distribution** of the test statistic under H_0 — $X \sim B(n, p)$ where n is the sample size.
6. State the **significance level**, α — you'll usually be given this.

   The **significance level** of a test (α) determines **how unlikely** your data needs to be under the null hypothesis (H_0) before you reject H_0.

   E.g. a significance level of $\alpha = 0.05$ would mean that you would **only** reject H_0 if your observed data fell into the **most extreme 5%** of possible outcomes.

Example **Cleo wants to test whether a coin is more likely to land on heads than tails. She plans to flip it 15 times and record the results. Write down suitable null and alternative hypotheses. Define the test statistic, X, and give its probability distribution under the null hypothesis.**

- Define your population parameter. Let p be the probability of the coin landing on heads.
- The null hypothesis will be that the coin is unbiased. $\mathbf{H_0: p = 0.5}$
- Cleo believes the coin is more likely to land on heads. $\mathbf{H_1: p > 0.5}$
- Each 'heads' is a success. **X = the number of heads in the sample of 15 throws**
- Under H_0, $p = 0.5$ and $n = 15$. $\mathbf{X \sim B(15, 0.5)}$

Exercise 2.1

MODELLING

Q1 For each hypothesis test described below, write down suitable null and alternative hypotheses, define the test statistic, X, and give its probability distribution under the null hypothesis.

a) Callie believes a 10-sided spinner is biased towards landing on 7. She plans to test this by spinning the spinner 50 times and recording the results.

b) The probability of being stopped at a particular set of traffic lights is thought to be 0.25. Eli thinks he is less likely to be stopped. He passes the lights once a day for 2 weeks and records whether or not he has to stop.

c) A taxi company's drivers get lost on average on 1 in every 40 journeys. The company employs some new drivers and wants to test whether this proportion has changed, using a random sample of 100 journeys.

d) A school health team checks teenagers for the presence of an antibody before vaccinating them. Usually 35% of teenagers have the antibody present. The team is to select a random sample of 40 teenagers from a remote Scottish island where they think that this proportion may be different.

e) Lucy believes that only 50% of students in her school will have seen a particular film. Rahim thinks that a higher proportion of students will have seen the film, so asks a random sample of 30 students.

f) Each day, the probability that a cat catches a mouse is 0.7. The cat's owner has put a bell on its collar and wants to test if it now catches fewer mice. She records whether a mouse is caught every day for 3 weeks after attaching the bell.

Exercise 1.1

Q1 **a)** The probability that a seed germinates.

b) 0.9

c) Call the probability p.
Then **(i)** H_0: $p = 0.9$ **(ii)** H_1: $p > 0.9$

d) One-tailed

Q2 Let p be the probability that a randomly selected resident knows about the grants. Then H_0: $p = 0.16$ and H_1: $p > 0.16$.

Q3 Let p be the proportion of customers who buy a jar of chilli chutney. Then H_0: $p = 0.03$ and H_1: $p \neq 0.03$.

Q4 Let p be the probability that a randomly selected gym member watches Australian soaps. Then H_0: $p = 0.4$ and H_1: $p > 0.4$.

Exercise 2.1

Q1 **a)** Let p be the probability of the spinner landing on 7.
Then H_0: $p = 0.1$ and H_1: $p > 0.1$.
Let X be the number of times the spinner lands on 7 in 50 spins.
Then under H_0, $X \sim B(50, 0.1)$.

b) Let p be the probability of Eli being stopped at the traffic lights. Then H_0: $p = 0.25$ and H_1: $p < 0.25$.
Let X be the number of times Eli is stopped in 2 weeks. Then under H_0, $X \sim B(14, 0.25)$.

c) Let p be the probability that a driver gets lost on any journey. Then H_0: $p = 0.025$ and H_1: $p \neq 0.025$.
Let X be the number of journeys where the driver gets lost in the sample of 100. Then under H_0, $X \sim B(100, 0.025)$.

d) Let p be the probability that a randomly selected teenager from the Scottish island has the antibody present. Then H_0: $p = 0.35$ and H_1: $p \neq 0.35$. Let X be the number of teenagers in the sample who have the antibody present. Then under H_0, $X \sim B(40, 0.35)$.

e) Let p be the probability that a randomly selected student has seen the film. Then H_0: $p = 0.5$ and H_1: $p > 0.5$. Let X be the number of students in the sample who have seen the film. Then under H_0, $X \sim B(30, 0.5)$.

f) Let p be the probability that a mouse is caught on any day.
Then H_0: $p = 0.7$ and H_1: $p < 0.7$.
Let X be the number of days a mouse is caught in the sample of 21. Then under H_0, $X \sim B(21, 0.7)$.